Algebra and Trigonometry

Lines

Slope Intercept Form with slope m and y -intercept b:

$$y = mx + b$$

Point-Slope Form of line through a point P = (x_1, y_1):

$$y - y_1 = m(x - x_1)$$

Slope

$$\frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

For Equations $y_1 = m_1 x + b$ and $y_2 = m_2 x + b$

Lines are parallel if they have the same slope: $m_1 = m_2$

Lines are perpendicular if they their slopes are negative reciprocals of each other:

$$m_1 = \frac{-1}{m_2}$$

Distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of the line connecting two points (x_1, y_1) and (x_2, y_2) is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Conic Sections

Equation of a Circle with center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

Equation of an Ellipse with center (h, k) and radii a and b:

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Equation of a Hyperbola with center (h, k) and radii a and b:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Equation of a Parabola with center (h, k):

$$(y - k)^2 = 4p\,(x - h)\ or\ (x - h)^2 = 4p\,(y - k)$$

Fractions

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

$(\frac{a}{b})^n = \frac{a^n}{b^n}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Exponent Rules

$x^a x^b = x^{a+b}$

$x^{-a} = \frac{1}{x^a}$

$(xy)^a = x^a y^a$

$(x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{a-b}$

$x^{1/a} = \sqrt[a]{x}$

$\sqrt[a]{xy} = \sqrt[a]{x}\sqrt[a]{y}$

$x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$

$x^0 = 1,\ x \neq 0$

Logarithm Rules

$\log(x) = \log_{10}(x)$

$\ln(x) = \log_e(x)$

$\ln(ab) = \ln(a) + \ln(b)$

$\ln(a^b) = b\ln(a)$

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$\ln\left(\frac{1}{x}\right) = -\ln(x)$

$\ln(e) = 1$

Special Factorizations and Binomial Theorem

Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$

Perfect Squares: $a^2 + 2ab + b^2 = (a + b)^2$, $a^2 - 2ab + b^2 = (a - b)^2$

Sum of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Perfect Cubes: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Binomial Theorem:

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \binom{n}{k}a^{n-k}b^k + \cdots + nab^{n-1} + b^n$$

$$where \binom{n}{k} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k(k-1)(k-2)\ldots(2)(1)} = \frac{n!}{k!\,(n-k)!}$$

Quadratic Formula

If $ax^2 + bx + c = 0, then\ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometry

Area of a Triangle: $A = \frac{1}{2}bh$

Area of a Circle: $A = \pi r^2$

Area of a Rectangle: $A = bh$

Area of a Trapezoid: $A = \frac{b_1+b_2}{2}h$

Perimeter of a Triangle: $A = a + b + c$

Perimeter of a Circle: $P = \text{Circumference} = 2\pi r = \pi d$

Perimeter of a Rectangle: $A = 2(b + h)$

Volume of a Sphere: $V = \frac{4\pi}{3}r^3$

Volume of a Cylinder: $V = \pi r^2 h$

Volume of a Rectangular Prism: $V = bhd$

Volume of a Cone: $V = \frac{\pi}{3} r^2 h$

Volume of a Prism: $V = Area_{face} \cdot depth$

Surface Area of a Sphere: $SA = 4\pi r^2$

Surface Area of a Cylinder: $SA = 2\pi r(r + h)$

Surface Area of a Rectangular Prism: $SA = 2(bh + bd + hd)$

Surface Area of a Cone: $SA = \pi r\sqrt{r^2 + h^2}$

Trigonometry

Angles

π radians = 180°

To convert x degrees to radians, $x° \cdot \frac{\pi}{180°}$

To convert x radians to degrees, $x \cdot \frac{180°}{\pi}$

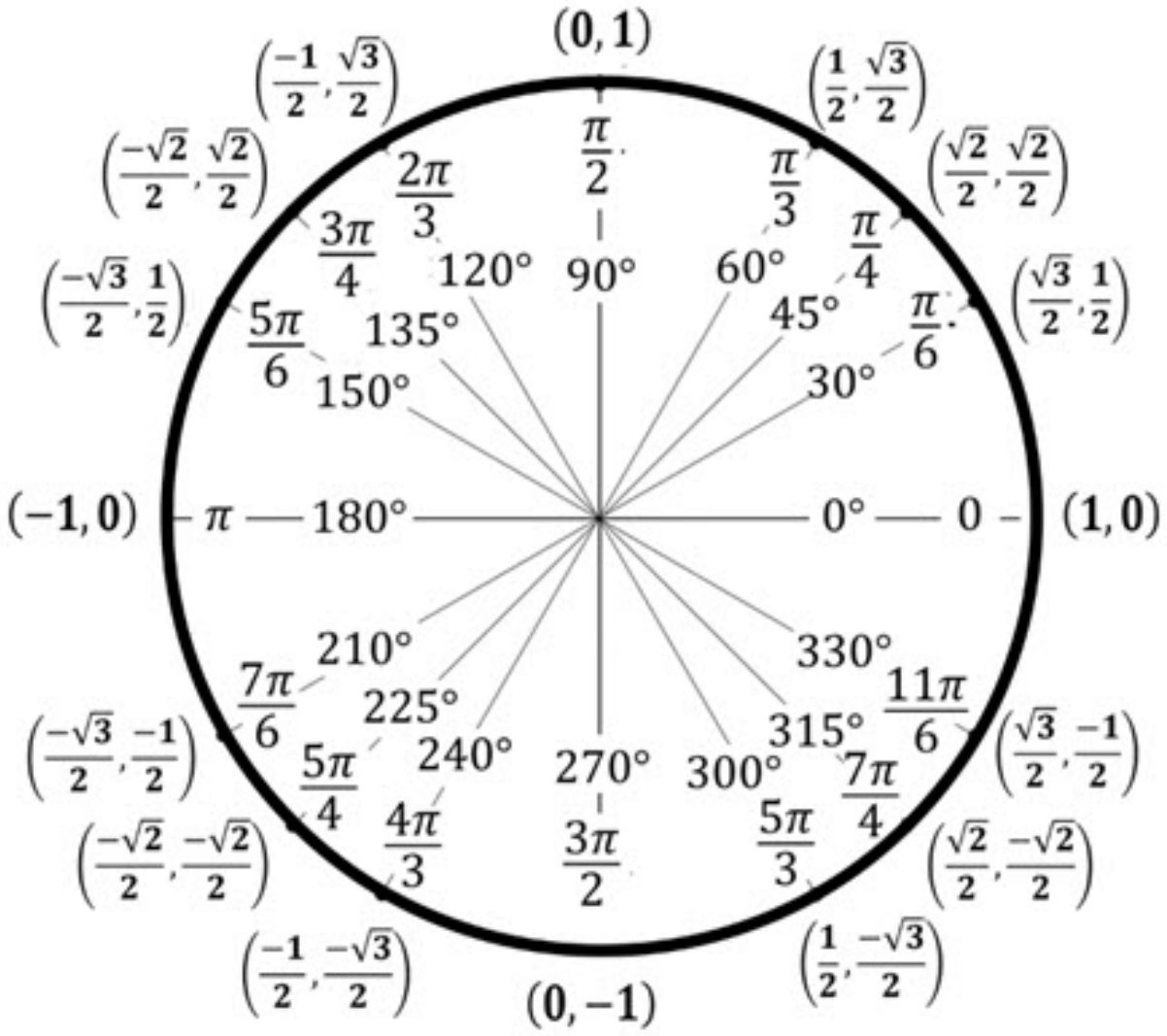
Right Angle Definitions

$$sin\theta = \frac{opposite}{hypotenuse} \qquad cos\theta = \frac{adjacent}{hypotenuse}$$

$$tan\theta = \frac{sin\theta}{cos\theta} = \frac{opposite}{adjacent} \qquad cot\theta = \frac{cos\theta}{sin\theta} = \frac{adjacent}{opposite}$$

$$sec\theta = \frac{1}{cos\theta} = \frac{hypotenuse}{adjacent} \qquad csc\theta = \frac{1}{sin\theta} = \frac{hypotenuse}{opposite}$$

Unit Circle



Trigonometric Identities

$$sin^2\theta + cos^2\theta = 1$$

$$r^2sin^2\theta + r^2cos^2\theta = r^2$$

$$1 + tan^2\theta = sec^2\theta$$

$$1 + cot^2\theta = csc^2\theta$$

$$\sin(-\theta) = -sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -tan\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right)=cos\theta \qquad \tan\left(\frac{\pi}{2}-\theta\right)=cot\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=sin\theta \qquad \sin(\theta+2\pi)=sin\theta$$

$$\cos(\theta+2\pi)=cos\theta$$

$$\tan(\theta+\pi)=tan\theta$$

Law of Sines: $\frac{SinA}{a}=\frac{SinB}{b}=\frac{SinC}{c}$

Law of Cosines: $c^2=a^2+b^2-2ab\cos(C)$

Addition and Subtraction Formulas

$$\sin(a+b)=sina\,cosb+cosa\,sinb$$

$$\sin(a-b)=sina\,cosb-cosa\,sinb$$

$$\cos(a+b)=cosa\,cosb-sina\,sinb$$

$$\cos(a-b)=cosa\,cosb+sina\,sinb$$

$$\tan(a+b)=\frac{tana+tanb}{1-tana\,tanb}$$

$$\tan(a-b)=\frac{tana-tanb}{1+tana\,tanb}$$

Angle Formulas

$$\cos(2a)=\ cos^2a-sin^2a=2cos^2a-1=1-2sin^2a$$

$$\sin(2a)=2sina\,cosa$$

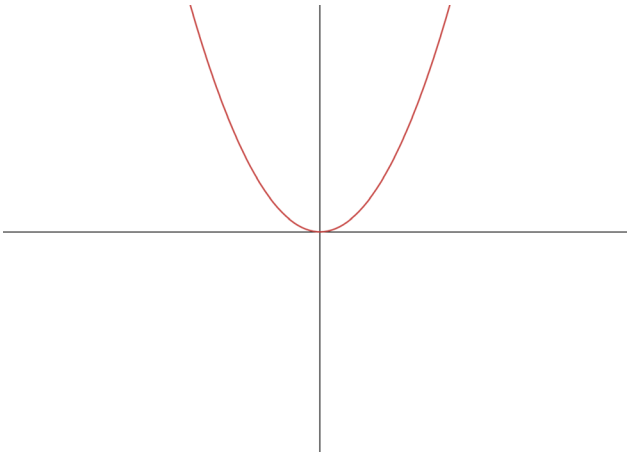
$$\tan(2a)=\frac{2tana}{1-tan^2a}$$

$$sin^2a=\frac{1-\cos(2a)}{2}$$

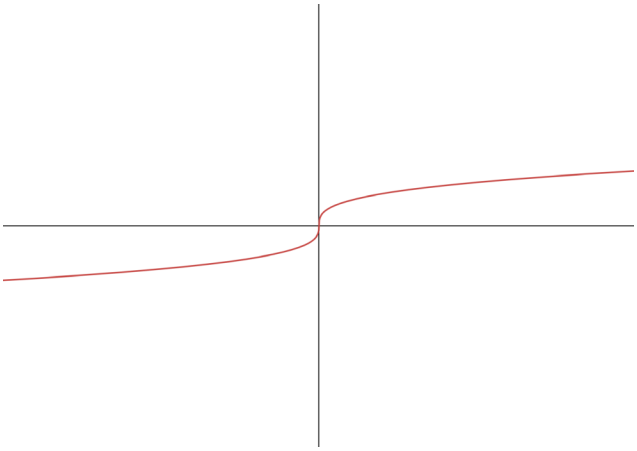
$$cos^2a=\frac{1+\cos(2a)}{2}$$

Parent Functions

$f(x) = x^n, n > 0$ *and even*



$f(x) = x^{1/n} = \sqrt[n]{x}, n > 0$ *and odd*



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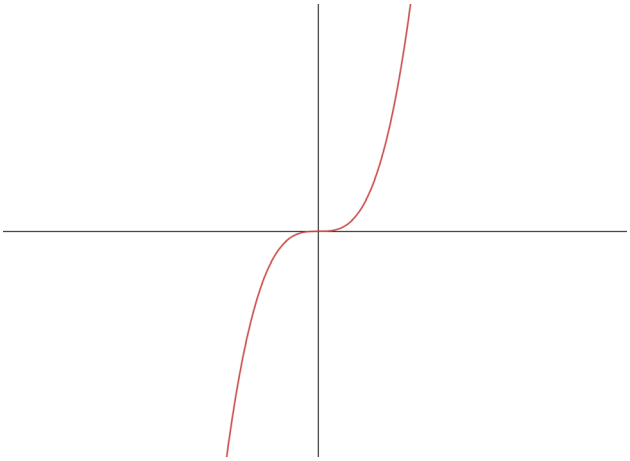
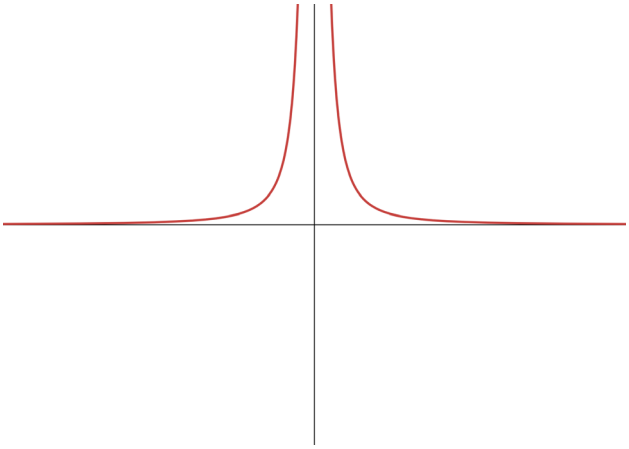
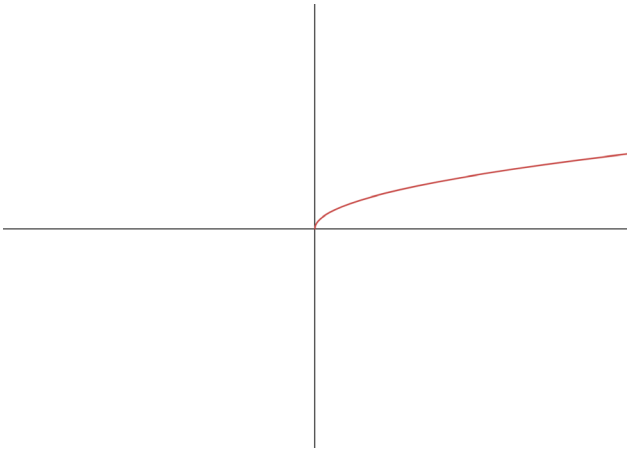


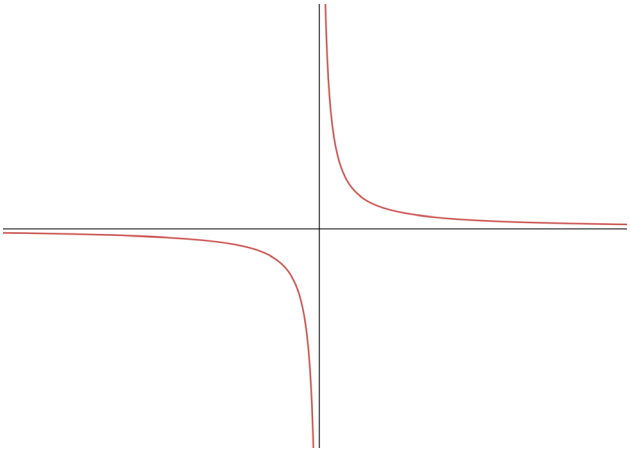
$f(x) = x^{-n} = \frac{1}{x^n}, n$ *even and positive*



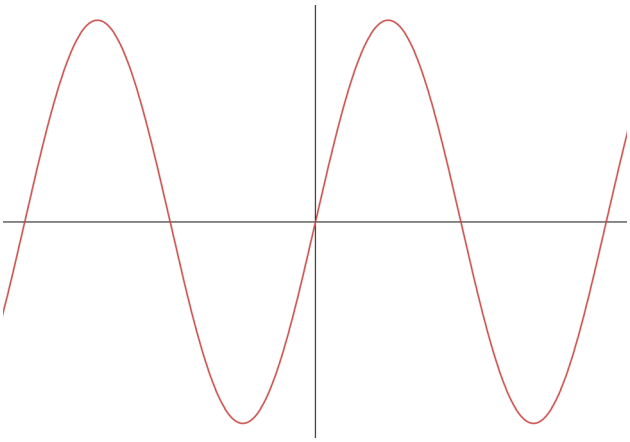
$f(x) = x^{1/n} = \sqrt[n]{x}, n > 0$ *and even*



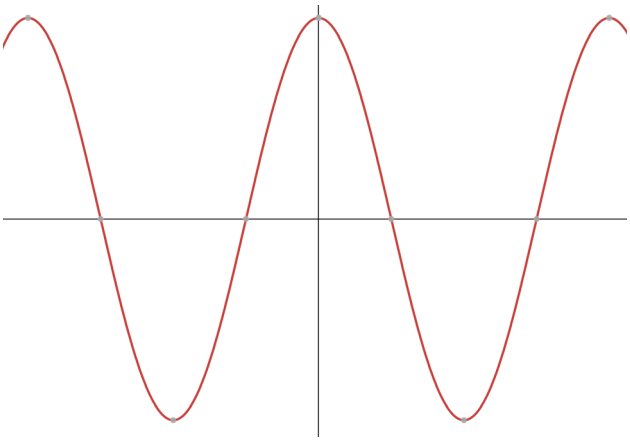
$f(x) = x^{-n} = \frac{1}{x^n}, n\ odd\ and\ positive$



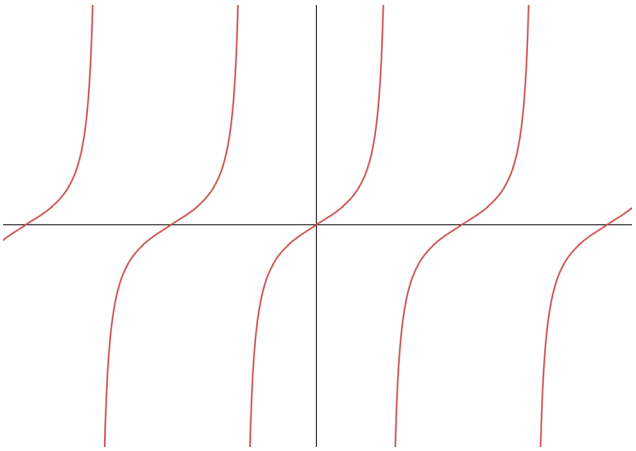
$f(x) = sinx$



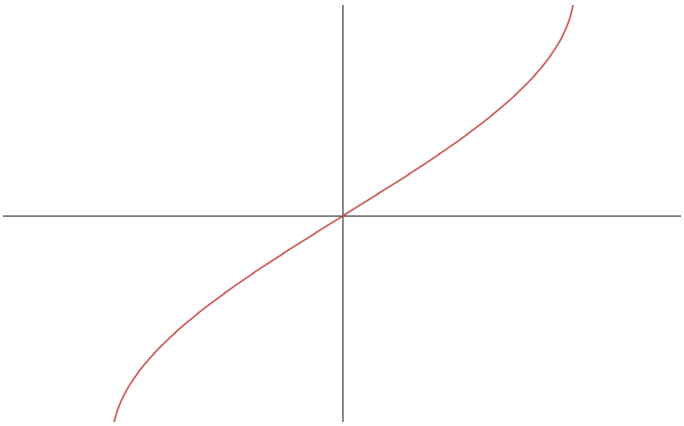
$f(x) = cosx$



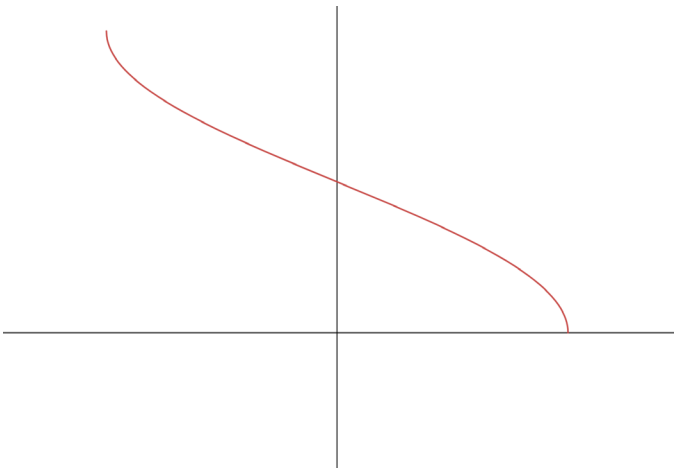
$f(x) = tanx$



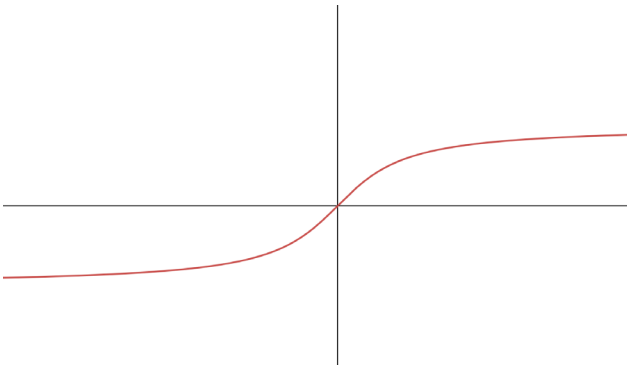
$f(x) = \arcsin(x), -1 \leq x \leq 1$



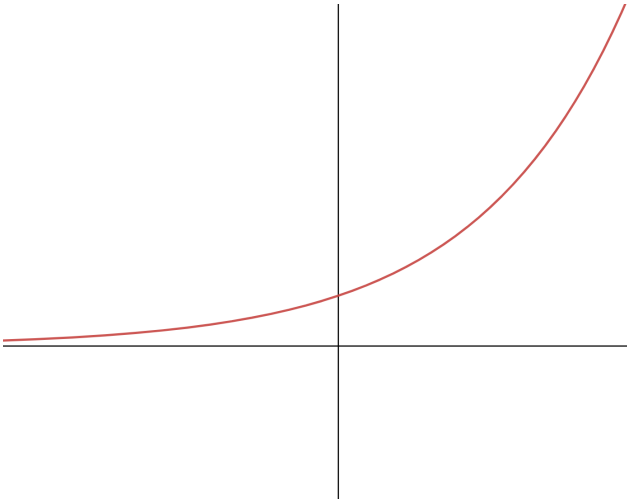
$f(x) = \arccos(x), -1 \leq x \leq 1$



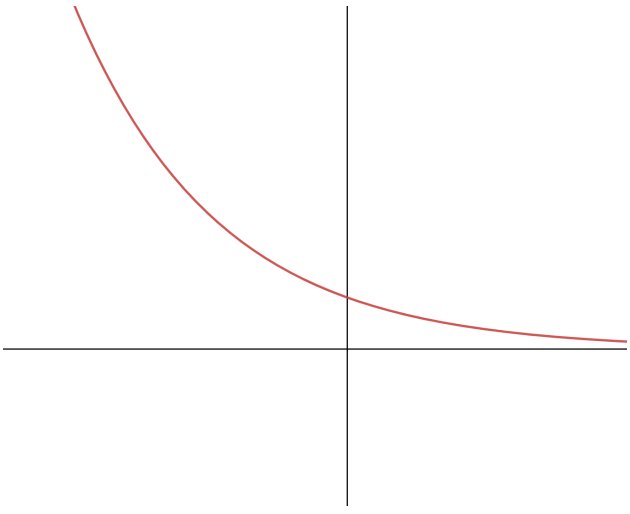
$f(x) = \arctan(x), -\infty < x < \infty$



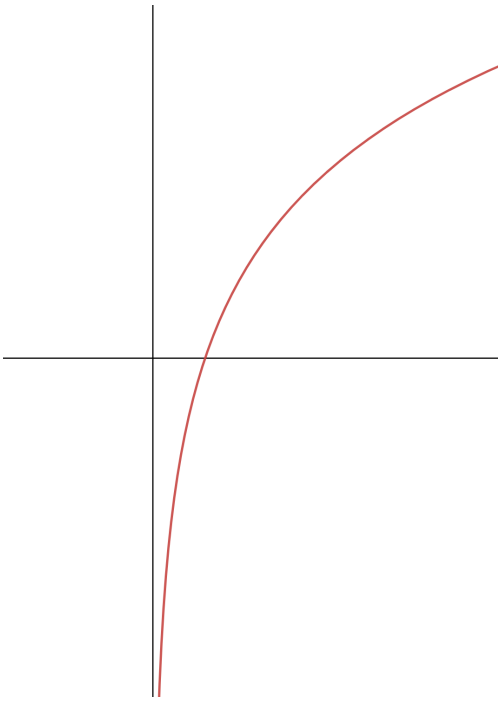
$f(x) = a^x, a > 1$



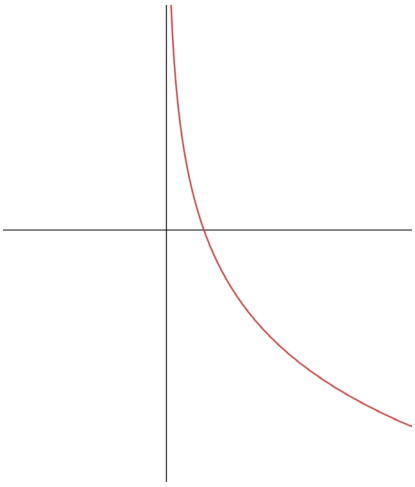
$f(x) = a^x, \ 0 < a < 1$



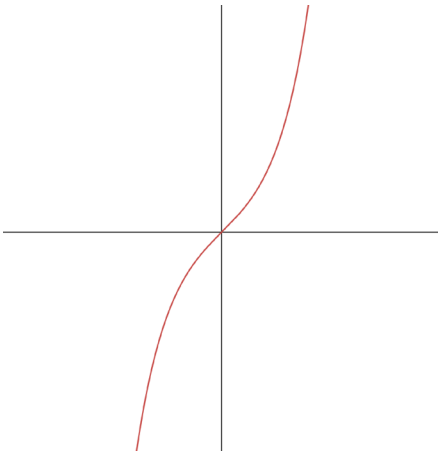
$f(x) = \log_a x, \ a > 1, \ x > 0$



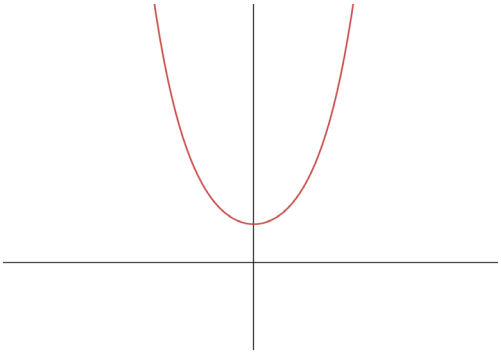
$f(x) = \log_a x, \ 0 < a < 1, \ x > 0$



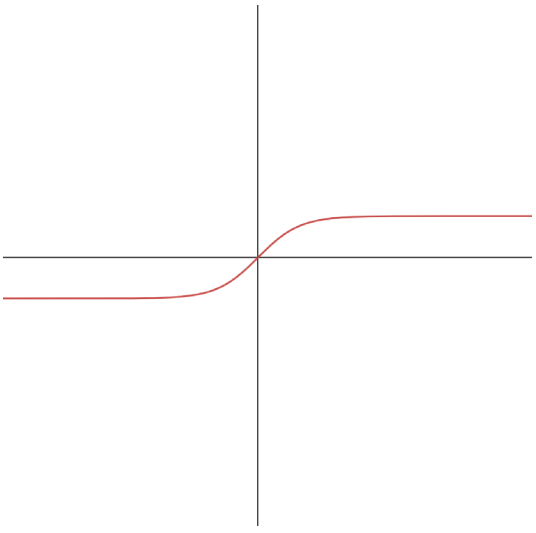
$f(x) = \sinh(x)$



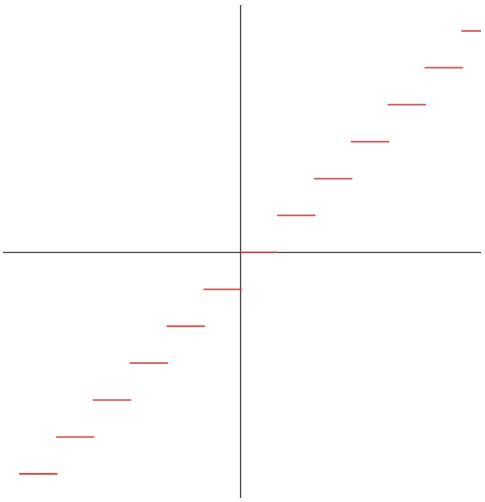
$f(x) = \cosh(x)$



$f(x) = \tanh(x)$



$f(x) = \lfloor x \rfloor$



$f(x) = |x|$