Calculus II Homework

August 9, 202

Pg. 310

61. Compute the area under the graph as a limit.

$f(x) = x^3 \quad [0,1]$

$\Delta x = \frac{1-0}{N} = \frac{1}{N}$

$\lim_{N\to\infty}$ $\lim_{N\to\infty}$

$R_N = \frac{1}{N} \sum_{1}^{N} f\left(0+\frac{i}{N}\right)^3 \rightarrow R_N = \frac{1}{N} \sum_{1}^{N} \frac{i^3}{N^3}$

$\lim_{N\to\infty}$

$R_N = \frac{1}{N}\left(\frac{1}{N^3} \cdot \frac{N^4}{4}\right) \rightarrow \frac{1}{\cancel{N}}\left(\frac{\cancel{N}}{4}\right) \rightarrow \boxed{R_N = \frac{1}{4} \text{ units}^2}$

✓ 65. Compute the area under the graph as a limit.

$f(x) = 3x^2 - x + 4 \quad [1,5]$

$\Delta x = \frac{5-1}{N} = \frac{4}{N}$

$R_N = \lim_{N\to\infty} \frac{4}{N} \sum_{1}^{N} f\left(1+\frac{4i}{N}\right)$

$R_N = \lim_{N\to\infty} \frac{4}{N} \cdot \left(3\left(1+\frac{4i}{N}\right)^2 - \left(1+\frac{4i}{N}\right) + 4\right)$

$R_N = \lim_{N\to\infty} \frac{4}{N} \cdot \left(3\left(1+\frac{8i}{N}+\frac{16i^2}{N^2}\right) - 1 - \frac{4i}{N} + 4\right)$

$R_N = \lim_{N\to\infty} \frac{4}{N} \cdot \left(3 + \frac{24i}{N} + \frac{48i^2}{N^2} - 1 - \frac{4i}{N} + 4\right)$

$R_N = \lim_{N\to\infty} \frac{4}{N} \cdot \left(6 + \frac{20i}{N} + \frac{48i^2}{N^2}\right) = \frac{4}{N} \cdot \left(6N + \frac{20}{\cancel{N}} \cdot \frac{N^2}{2} + \frac{48}{\cancel{N^2}} \cdot \frac{N^3}{3}\right)$

$R_N = \frac{4}{N} \cdot (6N + 10N + 16N) = 24 + 40 + 64 = 128 \text{ units}^2$

Calculus II Homework (cont.)

67. Compute the area under the graph as a limit.

$f(x) = x^2 \quad [2, 4]$

$\Delta x = \frac{4-2}{N} = \frac{2}{N}$

$$R_N = \frac{2}{N} \cdot \sum_{1}^{N} f\left(2 + \frac{2i}{N}\right) = \frac{2}{N} \cdot \sum_{1}^{N} \left(2 + \frac{2i}{N}\right)^2$$

$$R_N = \frac{2}{N} \cdot \sum_{1}^{N} \left(4 + \frac{8i}{N} + \frac{4i^2}{N^2}\right) = \frac{2}{N} \cdot \left(\sum_{1}^{N} 4 + \sum_{1}^{N} \frac{8i}{N} + \sum_{1}^{N} \frac{4i^2}{N^2}\right)$$

$$R_N = \frac{2}{N} \cdot \left(4N + \frac{8}{N} \cdot \frac{N^2}{2} + \frac{4}{N^2} \cdot \frac{N^3}{3}\right) = \frac{2}{N} \cdot \left(4N + 4N + \frac{4}{3}N\right)$$

$$R_N = \frac{2}{\cancel{N}} \cdot \frac{28\cancel{N}}{3} = \boxed{\frac{56}{3} \text{ OR } 18.\overline{6} \text{ units}^2}$$