116 # The Fundamental Theorem of Calc, 8/12/2

"Evaluating Area"

$\lim_{N \to \infty} \sum 1 = n$, $L \sum i = \frac{n^2}{2}$, $L \sum i^2 = \frac{n^3}{3}$, etc.

$\frac{d}{dt} x(t) = v(t)$, $\frac{d}{dt} v(t) = a(t)$

$\int a(t)\, dt = v(t)$ $\int v(t)\, dt = x(t)$

The Integral

$\int f(x)\, dx = F(x) + C$

Have Both

$f(x)$ = integrand dx = differential $F(x)$ = antiderivative C = constant of integration

$\int_a^b f(x)\, dx = F(b) - F(a)$ ("the area of $f(x)$ from a to b" = Antiderivative of b - ant. of a

1. So, $\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$ "Integrals & derivatives cancel."

2. This allows us to do transforms.

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3. $\int_a^a f(x)\,dx = 0$

Indefinite Integrals

Ex. $\int 3x + 7dx \longrightarrow \frac{3x^2}{2} + \frac{7x^1}{1} + c = \frac{3x^2}{2} + 7x + c$

Antiderivative Rules

Power Rule: $\int x^n dx = \frac{x^{n+1}}{n+1}$

Trig Rules: $\int \sin(x)\,dx = -\cos(x)$ $\int \cos(x)\,dx = \sin(x)$

$\int b^x dx = \frac{b^x}{\ln(b)}$

Definite Integrals

$\int_1^5 2x+9\,dx \rightarrow \frac{2x^2}{2} + \frac{9x^1}{1} \rightarrow x^2+9x \Big|_1^5$

but $F(b) - F(a) \rightarrow 5^2 + 9(5) + 9$