

Pascal's Triangle & Binomial Expansion

$$\begin{array}{ccccccc}
 & & & & & & \\
 & & & & & & 1 \\
 & & & & & 1 & 2 & 1 \\
 & & & & 1 & 3 & 3 & 1 \\
 & & & 1 & 4 & 6 & 4 & 1 \\
 & & 1 & 5 & 10 & 10 & 5 & 1 \\
 & 1 & 6 & 15 & 20 & 15 & 6 & 1 \\
 & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 \\
 & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1
 \end{array}$$

Binomial Expansion

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} a^0 b^n$$

$$(x+3)^5$$

$$\begin{aligned}
 & \binom{5}{0} x^5 3^0 + \binom{5}{1} x^4 3^1 + \binom{5}{2} x^3 3^2 + \binom{5}{3} x^2 3^3 + \binom{5}{4} x^1 3^4 + \binom{5}{5} x^0 3^5 \\
 & x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243 \\
 & (2x-y^3)^4
 \end{aligned}$$

$$\begin{aligned}
 & \binom{4}{0} (2x)^4 (-y^3)^0 + \binom{4}{1} (2x)^3 (-y^3)^1 + \binom{4}{2} (2x)^2 (-y^3)^2 + \binom{4}{3} (2x)^1 (-y^3)^3 + \binom{4}{4} (2x)^0 (-y^3)^4 \\
 & 16x^4 - 32x^3 y^3 + 24x^2 y^6 - 8x y^9 + y^{12}
 \end{aligned}$$

Composition

A composite function is a function with another function as its input

$$(f \circ g)(x) = f(g(x))$$

Usually $f(g(x)) \neq g(f(x))$

$$\{x \mid f(x) = x^2 + 1 \quad g(x) = \sqrt{x}\}$$

$$f(g(x)) = \sqrt{x}^2 + 1 = x + 1$$

$$g(f(x)) = \sqrt{x^2 + 1} \quad x \geq 0$$

$$\text{Ex } f(x) = 2x^3 + 3x - 4 \quad g(x) = 2^x$$

$$f(g(x)) = 2(2^x)^3 + 3(2^x) - 4$$

$$g(f(x)) = 2^{2x^3 + 3x - 4}$$