5.6 Substitution August 13

- is kind of a reverse chain rule

recall

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$\int g' \cdot f(g(x))\,dx = F(g(x)) + C$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{dy}{dx}$

$\int 2x \cdot \cos(x^2)dx$] chain rule derivative

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int 2x\cos(u) \cdot \frac{du}{2x}$

$\int \cos(u) \cdot du$

$\sin(u) + C$

$\boxed{\sin(x^2) + C}$

$\int 5x^3 \sqrt{x^4+9}\,dx$

$u = x^4 + 9$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$dx = \frac{du}{4x^3}$

$\int 5x^3 \sqrt{u} \cdot \frac{du}{4x^3}$

$\int 5/4 \sqrt{u}\,du$

$\int 5/4\, u^{1/2}\,du$

$\int \frac{5/4\, u^{3/2}}{3/2} + C$

$\boxed{5/6(x^4+9)^{3/2} + C}$

Shortcut = $dx = \frac{du}{\text{derivative}}$

X $\int x\sin(x)dx$

X $\int (3x+1)\tan(x^2+x)dx$

✓ $\int \frac{\ln x}{x}\,dx$ $u = \ln x$ to solve

X $\int 4\sqrt{x^3-x}\,dx$

$\int \tan^3 x \sec^2 x\,dx$

$u = \tan x$

or

$u = \sec x$

$dx = \frac{du}{\sec^2 x}$

$\int u^3 \sec^2 x \frac{du}{\sec^2 x}$

$\int u^3 du$

$\frac{1}{4} u^4 + C$

$\boxed{\frac{1}{4}\tan^4(x) + C}$

$\int \frac{x}{x^2+1}\,dx$

$u = x^2 + 1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\int \frac{x}{u} \cdot \frac{du}{2x}$

$1/2 \int u^{-1} du$

$1/2 \ln u + C$

$\boxed{1/2 \ln(x^2+1) + C}$