ANTIDERIVITIVES + INTEGRALS

- $\frac{d}{dt}\ x(t) = v(t)$ · $\int v(t)\,dt = x(t)$
  $\frac{d}{dt}\ v(t) = a(t)$ $\int a(t)\,dt = x(t)$
- $\int f(x)\ dx = F(x) + c$ ← constant of integration
  integral differential antiderivative
  have to have both

★ essentially, antiderivatives & areas are the same thing

- THE FUNDAMENTAL THEOREM OF CALCULAS
  $\int_a^b f(x)\,dx = F(b) - F(a)$ – "the area of f(x) from a to b" = antiderivative of b − antiderivative of a

INTEGRALS II

1. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
   ↳ integrals & derivatives cancel out
2. this allows us to do Transforms – changing the variable
3. $\int_a^a f(x)\,dx = \varnothing$

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$\int \sin x\,dx = -\cos x$

$\int \cos x\,dx = \sin x$

$\int b^x\,dx = \frac{b^x}{\ln b}$

ex. INDEFINITE INTEGRALS:

ex $\int 3x + 7\,dx \rightarrow \frac{3x^2}{2} + 7x + c$

ex $\int 2^x + 3x^5 - 7x^{-2} + x^{1/2}\,dx \rightarrow \frac{2^x}{\ln 2} + \frac{3x^6}{6} - \frac{7x^{-1}}{-1} + \frac{1}{2}\cdot\frac{x^{3/2}}{3/2} + c \rightarrow \frac{2^x}{\ln 2} + \frac{1}{2}x^6 + 7x^{-1} + \frac{2}{3}x^{3/2} + c$

DEFINITE INTEGRALS

ex $\int_1^5 2x + 9\,dx \rightarrow x^2 + 9x\Big|_1^5 \rightarrow (5^2 + 9(5)) - (1^2 + 9(1)) \rightarrow 70 - 10 \rightarrow 60$

ex. $\int_{-2}^{3} x^3 + 1\,dx \rightarrow \frac{x^4}{4} + 1x\Big|_{-2}^{3} \rightarrow \left(\frac{3^4}{4} + 3\right) - \left(\frac{3^{-2}}{4} - 2\right) \rightarrow \frac{81}{4} + 3 - \frac{16}{4} + 2 \rightarrow 21.25$

SUBSTITUTION

- sort of like a reverse chainrule
  ↳ $\int g' \cdot f(g(x)) = F(g(x)) + c$
- RECALL $\frac{d}{dx}\ f(g(x)) = f'(g(x)) \cdot g'(x) \quad = \quad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{dy}{dx}$

ex $\int 2x \cdot \cos(x^2)\,dx \rightarrow \int 2x\cos(u)\cdot\frac{du}{2x} \rightarrow \int \cos(u)\,du \rightarrow \sin(u) + c \rightarrow \sin(x^2) + c$
↳ bc. $u = x^2 \rightarrow \frac{du}{dx} = 2x \rightarrow du = 2x\,dx \rightarrow dx = \frac{du}{2x}$

ex $\int 5x^3\sqrt{x^4 + 9}\ dx \rightarrow \int 5x^3 \cdot u^{1/2}\cdot\frac{du}{4x^3} \rightarrow \int \frac{5}{4}u^{1/2}\,du \rightarrow \frac{5}{4}\cdot\frac{2u^{3/2}}{3} + c \rightarrow \frac{5}{6}(x^4 + 9)^{3/2} + c$
↳ $u = x^4 + 9 \rightarrow \frac{du}{dx} = 4x^3 \rightarrow du = 4x^3 \cdot dx \rightarrow dx = \frac{du}{4x^3}$

TRIG SUBSTITUTION

- $\int \tan\theta\,d\theta \rightarrow \int \frac{\sin\theta}{\cos\theta}\,d\theta \rightarrow \int \frac{\sin\theta}{u}\cdot\frac{du}{-\sin\theta} \rightarrow \int -u^{-1}\,du \rightarrow -\ln|u| + c \rightarrow -\ln|\cos\theta| + c$
  $u = \cos\theta$ | $\rightarrow \ln|\cos\theta^{-1}| + c \rightarrow \ln|\sec\theta| + c$
  $d\theta = du/{-\sin\theta}$

DOUBLE SUBSTITUTION

- $\int x\sqrt{5x+1}\,dx \rightarrow \int x \cdot u^{1/2} \frac{du}{5} \rightarrow \int \frac{u-1}{5} u^{1/2} \cdot \frac{du}{5} \rightarrow \frac{1}{25} \int u^{3/2} - u^{1/2}\,du \rightarrow \frac{1}{25}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + c$

$u = 5x+1$

$dx = du/5$

$u-1 = 5x$

$x = \frac{u-1}{5}$

$\rightarrow 2\left(\frac{(5x+1)^{5/2}}{125} - \frac{(5x+1)^{3/2}}{75}\right) + c$