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## Energy in Simple Harmonic Motion

↳ Conservation of Energy: energy in the universe is the same at all times

- $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \rightarrow kA^2 = mv^2 + kx^2 \rightarrow kA^2 - kx^2 = mv^2 \rightarrow v^2 = \frac{kA^2 - kx^2}{m}$

$v = \sqrt{\frac{kA^2 - kx^2}{m}} \rightarrow v = \sqrt{\frac{k(A^2 - x^2)}{m}} \rightarrow v = \sqrt{\frac{k}{m}\left(1 - \frac{x^2}{A^2}\right)A^2} \rightarrow \boxed{v = A\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{A^2}\right)}}$ current distance / max distance

- Max Velocity: $v_{max} = A\sqrt{\frac{k}{m}\left(1 - \frac{0}{A^2}\right)} \rightarrow \boxed{v_{max} = A\sqrt{\frac{k}{m}}}$

Period $= 2\pi\sqrt{\frac{m}{k}} \rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

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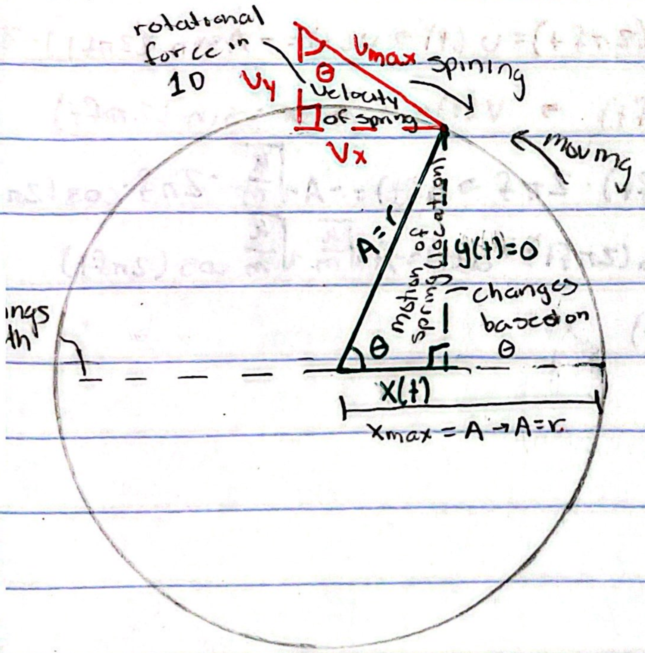
## The Period + Sinusoidal Nature of SHM




Because $\triangle$ is similar to $\triangle$ by 3 identical angles $\frac{v_x}{v_{max}} = \frac{\sqrt{A^2 - x^2}}{A}$ $\left(\frac{opp}{hyp}\right)$

$v_x = v_{max}\frac{\sqrt{A^2 - x^2}}{A} \rightarrow v_x = v_{max}\sqrt{\frac{A^2 - x^2}{A^2}}$

$\boxed{v_x = v_{max}\sqrt{1 - \frac{x^2}{A^2}}}$

Circumfrence $= 2\pi A \rightarrow d = r \cdot t \rightarrow 2\pi A = v_{max} \cdot T$

$\boxed{v_{max} = \frac{2\pi A}{T}}$ OR $\boxed{T = \frac{2\pi A}{v_{max}}}$ OR $\boxed{f = \frac{v_{max}}{2\pi A}}$

$\underbrace{\frac{1}{2}kA^2}_{\text{Max PE}} = \underbrace{\frac{1}{2}mv_{max}^2}_{\text{Max KE}} \rightarrow kA^2 = mv_{max}^2 \rightarrow A = m\frac{v_{ax}^2}{A^2}$

$\frac{k}{m} = \frac{v_{max}^2}{A^2} \rightarrow \sqrt{\frac{k}{m}} = \frac{v_{max}}{A} \rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

Big Idea: The springing does not depend on the stretchiness ($\Delta x$)

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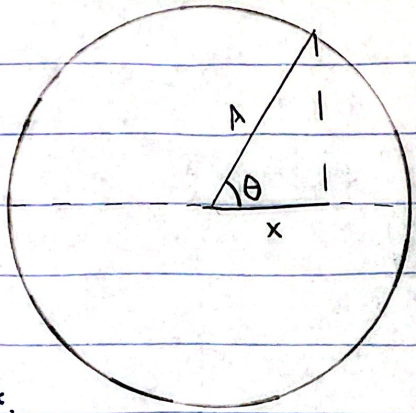
## Sinusoidal Nature of SHM




$x(t)$ = springs location (stretch) at time $t$

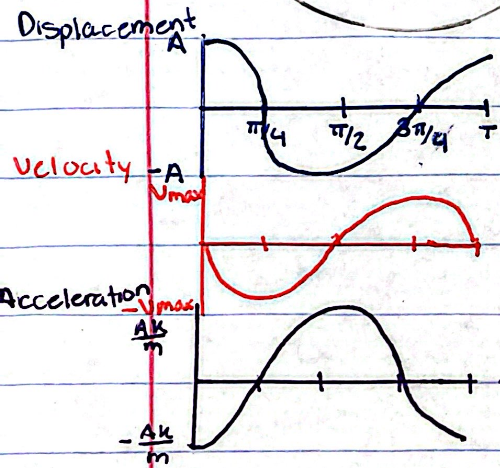
$\cos\theta = \frac{x(t)}{A}$ so $x(t) = A\cos(\theta)$

$\theta = \omega t$ where $\omega$ is the angular velocity

$x(t) = A\cos(\omega t) \rightarrow \omega = 2\pi f$ b/c $\omega = \frac{2\pi}{t}$

(how fast it spins in a circle)

so $x(t) = A\cos(2\pi f t) = A\cos\left(\frac{2\pi t}{T}\right)$




location is related to a cos function & a stretch

$\frac{d}{dt} x(t) = \frac{d}{dt} A\cos(2\pi f t) = v(t) \rightarrow v(t) = -A\sin(2\pi f t)\cdot 2\pi f$

$v = \overbrace{-2\pi f A}^{V_{max}} \cdot \sin(2\pi f t) \rightarrow V(t) = -V_{max}\cdot\sin(2\pi f t)$

$a(t) = -V_{max}\cos(2\pi f t)\cdot 2\pi f \rightarrow a(t) = -A\sqrt{\frac{k}{m}}\cdot 2\pi f\cdot\cos(2\pi f t)$

$a(t) = -V_{max}\sqrt{\frac{k}{m}}\cos(2\pi f t) \rightarrow a(t) = -A\sqrt{\frac{k}{m}}\sqrt{\frac{k}{m}}\cos(2\pi f t)$

$a(t) = -A\frac{k}{m}\cos(2\pi f t)$