# Integral Calculus

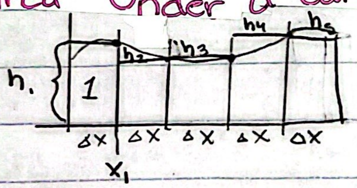
8/7/24

## Derivative Review

1. $y = x^2 \sin x \rightarrow 2x\sin x + x^2\cos x = y'$
2. $y = \sqrt{\tan x} \rightarrow y' = \frac{1}{2}(\tan x)^{-1/2} \cdot \sec^2 x$
3. $f(x) = 3x\,2^x \rightarrow f'(x) = 3(2^x) + 3x(2^x \cdot \ln(2))$  $\frac{d}{dx} b^x \rightarrow b^x \ln b$
4. $y = \frac{2x+7}{\ln x} \rightarrow y' = \frac{2\ln x - \frac{2x+7}{x}}{(\ln x)^2}$ ~OR~ $y = (2x+7)(\ln x)^{-1} \rightarrow y' = 2(\ln x^{-1}) - (2x+7)(\ln x)^{-2} \cdot \frac{1}{x}$
5. $y = \sqrt{\frac{x^2(\sin x)^{19}\sqrt{x}}{\ln x \cdot e^x}} \rightarrow \ln y = \ln\left(\frac{x^2(\sin x)^{19}\sqrt{x}}{\ln x \cdot e^x}\right)^{1/2} \rightarrow \ln(y) = \frac{1}{2}\left[\ln x^3 + \ln(\sin x)^{19} + \ln(x^{1/2}) - \ln(\ln x) - \ln(e^x)\right]$

$2\ln y = 3\ln x + 19\ln(\sin x) + \frac{1}{2}\ln x - \ln(\ln x) - x \rightarrow \cancel{2}\frac{y'}{\cancel{y}} = \left(\frac{3}{x} + \frac{19\cos}{\sin} + \frac{1}{2x} - \frac{1}{\ln x} \cdot \frac{1}{x} - 1\right)\frac{y}{2}$

## Area Under a Curve




$\Delta x$ = width of the rectangle

$h_1$ = height of rectangle 1  $f(0+\Delta x)$

$h_2 = f(0+2\Delta x)$

Estimate w $R_5$ $\rightarrow$ Area = width · height: $R_5 = \Delta x\,[f(\Delta x) + f(2\Delta x) + f(3\Delta x) + f(4\Delta x) + f(5\Delta x)]$

↳ $R_N = \Delta x \cdot \sum_{i}^{n} f(a + \Delta x\, i)$  $\Delta x = \frac{b-a}{N}$

- $N$: # of rectangles
- $\Delta x$: width
- $\sum_{i=1}^{N}$: add up all (function) from the $1^{st}$ to the $n^{th}$
- $f(a+\Delta x\, i)$: height of the $i^{th}$ rectangle
- $a$: initial x-value (starting point)
- $b$: final x-value (ending point)
- $i$: which # rectangle is the current one

Ex: Estimate AUC of $f(x) = x^2 + 2$ on $[2, 5]$ using $R_4$

$\Delta x = \frac{5-2}{4} \rightarrow \frac{3}{4}$  $R_4 = \frac{3}{4}\sum_{1}^{4} f(2 + \frac{3}{4}i) \rightarrow R_4 = \frac{3}{4}\sum_{1}^{4}(2 + \frac{3}{4}i)^2 + 2$

$R_4 = \frac{3}{4}\left[((2+\frac{3}{4})^2 + 2) + ((2+\frac{6}{4})^2 + 2) + ((2+\frac{9}{4})^2 + 2) + ((2+\frac{12}{4})^2 + 2)\right]$

$R_4 = \frac{3}{4}[9.5625 + 14.25 + 20.0625 + 27] \rightarrow R_4 = \frac{3}{4}(70.875) \rightarrow R_4 = 53.15625 \text{ units}^2$

## Evaluate Area Under a Curve

↳ $A = \lim_{N \to \infty} \Delta x \sum_{1}^{N} f(a + \Delta x\, i)$

Ex: Evaluate the AUC for $f(x) = x^2 + 2$ on $[2, 5]$

$$L \frac{3}{n} \sum_{1}^{n} \left(2 + \frac{3i}{n}\right)^2 + 2 \to L \frac{3}{n} \sum_{1}^{n} 4 + \frac{12i}{n} + \frac{9i^2}{n^2} + 2 \to L \frac{3}{n} \sum_{1}^{n} 6 + \frac{12i}{n} + \frac{9i^2}{n^2}$$

$$L \frac{3}{n} \left[ \sum_{1}^{n} 6 + \sum_{1}^{n} \frac{12}{n} i + \sum_{1}^{n} \frac{9}{n^2} i^2 \right] \to L \frac{3}{\cancel{n}} \left[ 6\cancel{n} + \frac{\cancel{12}^{6}}{\cancel{n}} \cdot \frac{n^{\cancel{2}}}{\cancel{2}} + \frac{\cancel{9}^{3}}{\cancel{n^2}} \cdot \frac{n^{\cancel{3}}}{\cancel{3}} \right]$$

$$3(6 + 6 + 3) \to 3(15) \to 45$$

Recall If $\lim_{n \to \infty}$

$\sum_{1}^{n} 1 = n$ $\quad n$

$\sum_{1}^{n} i = \frac{n^2 + \cancel{n}}{2}$ $\quad \frac{n^2}{2}$

$\sum_{1}^{n} i^2 \ \frac{2n^3 + \cancel{3n^2} + \cancel{n}}{6}$ $\quad \frac{n^3}{3}$

$\sum_{1}^{n} i^3 = \frac{n^4 + \cancel{2n^3} + \cancel{n^2}}{4}$ $\quad \frac{n^4}{4}$