11. Find the area between $y = e^x$ and $y = e^{2x}$ over [0, 1].

SOLUTION As the graph of $y = e^{2x}$ lies above the graph of $y = e^x$ over the interval [0, 1], the area between the graphs is

$$\int_0^1 (e^{2x} - e^x)\, dx = \left(\frac{1}{2}e^{2x} - e^x \right) \Big|_0^1 = \frac{1}{2}e^2 - e - \left(\frac{1}{2} - 1 \right) = \frac{1}{2}e^2 - e + \frac{1}{2}.$$

16.

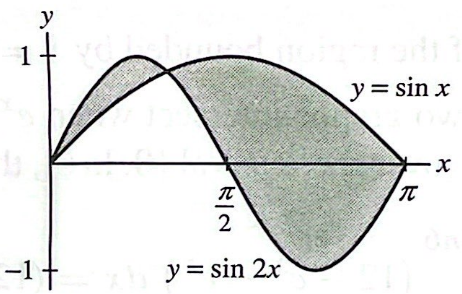

FIGURE 11

SOLUTION Setting $\sin x = \sin 2x$ yields $\sin x(2\cos x - 1) = 0$, so the points of intersection are $x = 0$, $x = \frac{\pi}{3}$ and $x = \pi$. Over the interval $[0, \frac{\pi}{3}]$, $y = \sin 2x$ is the upper curve but over the interval $[\frac{\pi}{3}, \pi]$, $y = \sin x$ is the upper curve. The area of the shaded region is then

$$\int_0^{\pi/3} (\sin 2x - \sin x)\, dx + \int_{\pi/3}^{\pi} (\sin x - \sin 2x)\, dx$$

$$= \left(-\frac{1}{2}\cos 2x + \cos x\right)\Bigg|_0^{\pi/3} + \left(-\cos x + \frac{1}{2}\cos 2x\right)\Bigg|_{\pi/3}^{\pi} = \frac{1}{4} + \frac{9}{4} = \frac{5}{2}.$$

17.

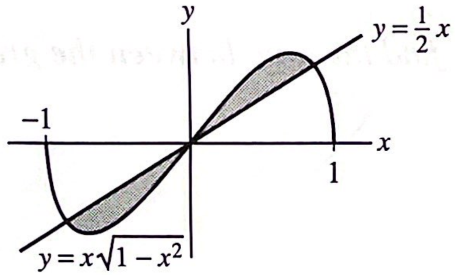

FIGURE 12

SOLUTION Setting $\frac{1}{2}x = x\sqrt{1-x^2}$ yields $x = 0$ or $\frac{1}{2} = \sqrt{1-x^2}$, so that $x = \pm\frac{\sqrt{3}}{2}$. Over the interval $[-\frac{\sqrt{3}}{2}, 0]$, $y = \frac{1}{2}x$ is the upper curve but over the interval $[0, \frac{\sqrt{3}}{2}]$, $y = x\sqrt{1-x^2}$ is the upper curve. The area of the shaded region is then

$$\int_{-\sqrt{3}/2}^{0} \left(\frac{1}{2}x - x\sqrt{1-x^2}\right) dx + \int_{0}^{\sqrt{3}/2} \left(x\sqrt{1-x^2} - \frac{1}{2}x\right) dx$$

$$= \left(\frac{1}{4}x^2 + \frac{1}{3}(1-x^2)^{3/2}\right)\Bigg|_{-\sqrt{3}/2}^{0} + \left(-\frac{1}{3}(1-x^2)^{3/2} - \frac{1}{4}x^2\right)\Bigg|_{0}^{\sqrt{3}/2} = \frac{5}{48} + \frac{5}{48} = \frac{5}{24}.$$