# 5.8 - Exponential Decay (& Growth)

$y = a \cdot b^x$ — *multiplying by the same number repeatedly

$0 < b < 1$, $b > 1$

"Growth Constant"

$y' = ky$ ← value

the growth rate

is proportional to

first-order differential equation

$$y' = ky$$

$$\frac{dy}{dt} = ky$$

$$dy = ky\,dt$$

$$\frac{\int dy}{y} = \frac{\int ky\,dt}{y}$$

$$\int \frac{dy}{y} = \int k\,dt$$

$$\ln|y| + C_1 = kt + C_2$$
$$-C_1 \qquad -C_1$$

$$\ln|y| = kt + C_2 - C_1$$

$$\ln|y| = kt + C_3$$

$$e^{\ln|y|} = e^{kt + C_3}$$

$$|y| = e^{kt} \cdot e^{C_3}$$

$$|y| = C_4 e^{kt}$$

$y = \pm C_4 e^{kt}$

$y = C_5 e^{kt}$

$\boxed{y = Ce^{kt}}$

$C$ → starting value

*all growth is related to e

Examples:

1. $k = 0.41 (hrs)^{-1}$

Assume 1,000 bacteria exist @ time $t = 0$

a) Find formula. $y' = 0.41y \rightarrow y = Ce^{0.41t}$

$y(0) = 1000 = Ce^{0.41 \cdot 0} = C \rightarrow \boxed{y = 1{,}000e^{0.41t}}$

b) How many after 5 hrs? $y = 1{,}000e^{0.41 \cdot 5} =$

$\boxed{7{,}767}$

c) When is it 10,000? -

$$\frac{10{,}000}{1{,}000} = \frac{1{,}000e^{0.41t}}{1{,}000}$$

$$10 = e^{0.41t}$$

ln ln

$$\frac{\ln(10)}{0.41} = \frac{0.41t}{0.41}$$

$$\boxed{t = 5.62 \text{ hrs}}$$

2. a) Write this as a diff. eq.

b) Find decay constant if $P(7) = 50$ mg, $P(0) = 450$ mg

c) When was 200 mg present?

$\boxed{P' = kP}$

$450 = Ce^{k(0)}$ ; $50 = Ce^{k(7)}$

$450 = C$

$50 = 450 e^{7k}$

$\frac{1}{9} = e^{7k}$

ln ln

$\frac{\ln(\frac{1}{9})}{7} = \frac{7k}{7}$

$\ln 1/9 = \ln 9^{-1}$

$a \ln b = \ln b^a$

$\ln 9^{-1/7} = k$

$\boxed{k = -0.31 (\text{hrs})^{-1}}$

$\frac{200}{450} = \frac{450 e^{-0.31t}}{450}$

$\frac{4}{9} = e^{-0.31t}$

ln ln

$\frac{\ln(\frac{4}{9})}{-0.31} = \frac{-0.31t}{-0.31}$

$\boxed{t = 2.6 \text{ hrs}}$