5.8

Compound Interest

$P(t) = P_0 \left(1 + \frac{r}{n}\right)^{tn}$

final value, initial value, # of compounds yrsly, rate of interest, increase, # of compounds

"start w/ \$1 @ 100% interest"

$P = 1\left(1 + \frac{1}{n}\right)^{nt}$

if $n=1$ $P(1) = (1+1)^1 = 2$

if $n=2$ $P(1) = \left(1+\frac{1}{2}\right)^2 = 2.25$

if $n=5$ $P(1) = \left(1+\frac{1}{5}\right)^5 = 2.48832$

if $n=\infty$ $P(1) = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \left(1+\frac{1}{\infty}\right)^\infty$

1^∞ → Indetermate

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$L\left(\frac{n+1}{n}\right)^n$

$\ln\left(L\left(\frac{n+1}{n}\right)^n\right) = L\ln\left(\frac{n+1}{n}\right)^n$

$\cancel{= Ln \ln\left(\frac{n+1}{n}\right)}$

$\cancel{= Ln \cdot L\ln\left(\frac{n+1}{n}\right)}$

$\cancel{= Ln \cdot L\ln(1)}$

$= 0 \cdot \infty$

TRY AGAIN

→ You gotta just plug in numbers!!! (unless he is lying)

$$\frac{d}{dx}x^x = x^x(\ln x + 1)$$

$$\frac{d}{dx}(f(x))^x$$

$y = f^x$

$\ln y = x\ln f$

$\frac{y'}{y} = \ln f + x\cdot\frac{f'}{f}$

$y' = \left(\ln f(x) + \frac{x f'(x)}{f(x)}\right) f(x)^x$

→ TRY THIS

if $n = 100,000,000,000$

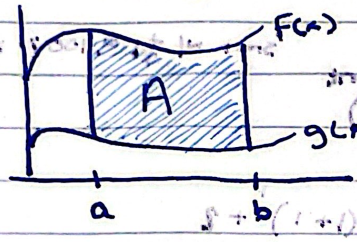
→ $P(1) = 2.718281828459045 23\ldots$

∴ e!!! wow!!

e is related to all population equations and also finance!

- e is so cool

6.1 Area Between Curves



$A = \text{Area } f(x) - \text{Area } g(x)$

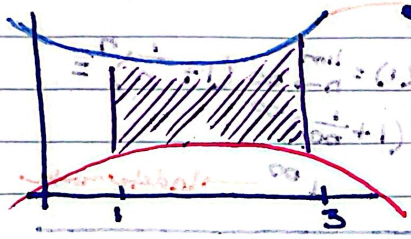
$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$= \int_a^b f(x) - g(x)\,dx$

Ex

find Area between

$f(x) = x^2 - 4x + 10$ and $g(x) = 4x - x^2$ on $[1, 3]$



Step 1: Draw graph
(can be put into calculator)

Step 2: top function - bottom function

$= \int_1^3 x^2 - 4x + 10 - (4x - x^2)\,dx$

$= \int_1^3 2x^2 - 8x + 10\,dx$

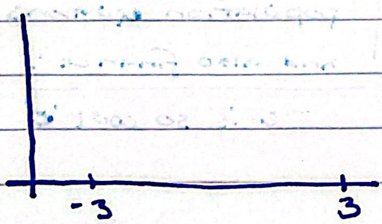
$= \frac{2x^3}{3} - 4x^2 + 10x \Big|_1^3$

$= [18 - 36 + 30] - [\frac{2}{3} - 4 + 10]$

$= 12 - \frac{20}{3} = \boxed{\frac{16}{3}}$

Ex

area between $y = 3x^2 + 12$ and $y = 4x + 4$ on $[-3, 3]$



$= \int_{-3}^{+3} 3x^2 + 12 - (4x + 4)\,dx$

$\int_{-3}^{3} 3x^2 - 4x + 8\,dx$

$x^3 - 2x^2 + 8x \Big|_{-3}^{3}$

$[27 - 18 + 24] - [-27 - 18 - 24]$

$54 + 48$

$\boxed{102}$